Hong Kong Mathematics Olympiad (2001 – 2002)

Heat Event (Group)

香港数学竞赛 (2001 – 2002)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 有糖果一袋分配给甲、乙、丙三人。甲、乙、丙三人依次所得的糖果数目的比是5∶4∶3。若把糖果重新分配给甲、乙、丙三人使其比依次为 7 : 6 : 5，则其中一人比原本所得的数目多了 40 粒，问此人原本所得的糖果数目。

   A bag of sweets is distributed to three persons A, B and C. The numbers of sweets obtained by A, B and C are in the ratios of $5:4:3$ respectively. If the sweets are re-distributed to A, B, C according to the ratios $7:6:5$ respectively, then one of them would get 40 more sweets than his original number. Find the original number of sweets obtained by this person.

2. 已知 a、b、c 为三个连续奇数且 $b^3 = 3375$，求 ac 的数值。

   Given that a, b, c are three consecutive odd numbers and $b^3 = 3375$, find the value of ac.

3. 设在直角坐标平面上不等式 $|x| + |y| \le 3$ 围出的多边形内面积为 p，求 p 的数值。

   Let p be the area of the polygon formed by the inequality $|x| + |y| \le 3$ in the Cartesian plane. Find the value of p.

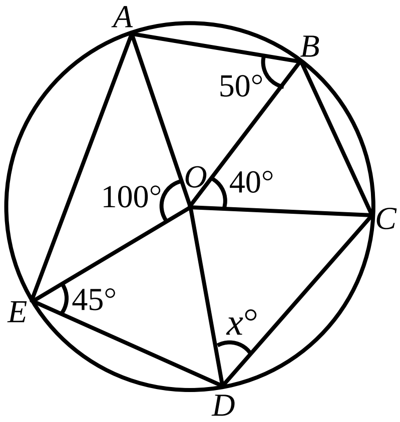
4. 求 $7^{2003} \div 100$ 的余数。

   Find the remainder of $7^{2003} \div 100$.

5. 如果实数 x、y 满足方程 $x^2 + y^2 + 3xy = 35$，求 xy 的最大值。

   If real numbers x, y satisfy the equation $x^2 + y^2 + 3xy = 35$, find the maximum value of xy.

6.



图一

Figure 1

如图一，点 A、B、C、D、E 位于以 O 为圆心的一个圆上。已知 $\angle DEO = 45°$， $\angle AOE = 100°$， $\angle ABO = 50°$， $\angle BOC = 40°$ 及 $\angle ODC = x°$，求 x 的数值。

In figure 1, points A , B , C , D , E are on a circle with centre at O . Given $\angle DEO = 45°$, $\angle AOE = 100°$, $\angle ABO = 50°$, $\angle BOC = 40°$, and $\angle ODC = x°$, find the value of x .

7. 将 20 个球放入 2 个袋中，每袋 10 个球，每袋的球分别标上数字 1 到 10，其中一个袋的球全为白色，另一个袋的球全为黑色。若从两个袋中任意各取一个球，求白球上的数字较黑球上的数字为大的概率。

20 balls are put into 2 bags with 10 balls in each bag. The balls in each bag are labeled numbers 1 to 10, all balls in one bag are white and all balls in the other bag are black. If one ball is drawn from each of two bags, find the probability that the number of the white ball is greater than that of the black ball.

8.

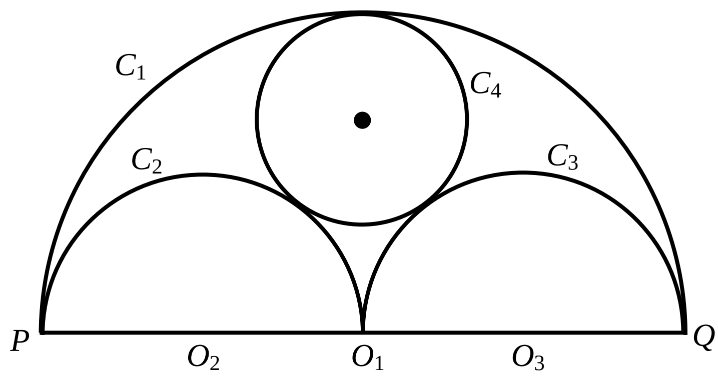


图 二

Figure 2

如图二， PQ、PO_1、O_1Q 分别是以 O_1、O_2、O_3 为圆心的半圆 C_1、C_2、C_3 的直径，圆 C_4 内切于半圆 C_1 及外切于半圆 C_2、C_3。 若 $PQ = 24$，求圆 C_4 的面积（取 $\pi = 3$）。

In Figure 2, PQ, PO_1, O_1Q are diameters of semi-circles C_1, C_2, C_3 with centres at O_1, O_2, O_3 respectively, and the circle C_4 touches C_1, C_2 and C_3. If $PQ = 24$, find the area of circle C_4. (Take $\pi = 3$).

9. 已知正整数 a 、 b 满足方程 $ab - a - b = 12$，求 ab 的值。

Given that a and b are positive integers satisfying the equation $ab - a - b = 12$, find the value of ab.

10. 已知三角形 ABC 中的 $\angle A$ 为一直角， $\sin^2 C - \cos^2 C = \frac{1}{4}$ ， $AB = \sqrt{40}$ 及 $BC = x$，求 x 的数值。

Given that $\angle A$ is a right angle in triangle ABC, $\sin^2 C - \cos^2 C = \frac{1}{4}$, $AB = \sqrt{40}$ and $BC = x$, find the value of x.